Exercise 1 (4 points)
Determine SVDs of the following matrices (by hand calculation):

(a) $\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$,  (b) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$,  (c) $\begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$,  (d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$,  (e) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Exercise 2 (4 points)
Suppose $A$ is an $m \times n$ matrix and $B$ is the $n \times m$ matrix obtained by rotating $A$ ninety degrees counter-clockwise on paper (not exactly a standard mathematical transformation!). Do $A$ and $B$ have the same singular values? Prove that the answer is yes or give a counterexample.

Exercise 3 (4 points)
Download the MATLAB program lintrans.m which draws the image of the unit sphere under a linear transformation with the matrix $A$ in 2D and 3D. Modify the program such that, given a $2 \times 2$ matrix $A$, it plots the right singular vectors $v_1$ and $v_2$ in the unit circle and also the left singular values $u_1$ and $u_2$ in the appropriate ellipse. Apply your program to the $2 \times 2$ matrices of exercise 1.

Exercise 4 (4 points)
Two matrices $A, B \in \mathbb{C}^{m \times m}$ are unitarily equivalent if $A = QBQ^*$ for some unitary $Q \in \mathbb{C}^{m \times m}$. Is it true or false that $A$ and $B$ are unitarily equivalent if and only if they have the same singular values?
General remarks

- Exercise sheets must be handed in individually, i.e., one exercise sheet per student! To get full points for your homework you are required to be able to present your results in the exercise group.

- Exercise sheets may be handed in just before the lecture.

- At the end of the term, there will be an oral examination (approx. 30 min.). To be admissible to the examination, at least 50% of the exercise points must have been obtained.

- Grades: You will get a grade $g_w$ based on your solutions to the exercises. The grade 1 will be given if you obtained at least 80% of the exercise points. Furthermore, you will get a grade $g_o$ in the oral examination. If you pass the oral examination ($g_o \leq 4$), your final grade will be the arithmetic mean of $g_w$ and $g_o$.

- All relevant information concerning the lecture and exercises can be found following the link at
  
  http://www-ai.math.uni-wuppertal.de/~dfritzsche/lehre/NLA_WSO8